Word Problems

Pernille B. Sunde

Pernille Pind

VIA University College

KU LEUVEN

INDEPENDENT RESEARCH FUND DENMARK

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Why?

Because it is difficult for a lot of students to solve word problems.

From the Danish curriculum: "The student can choose the appropriate type of calculation to solve simple everyday problems and create a simple arithmetic expression."

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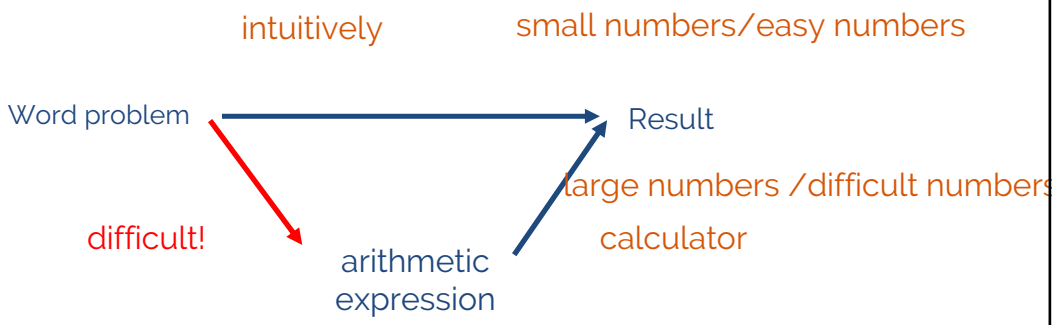
From word problem to result



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Simple Word Problems: 3 + 5

Combine:

Joe has 3 marbles. Tom has 5 marbles.
How many do they have together?

Change:

Joe had some marbles. Then he gave 5 marbles to Tom. Now Joe has 3 marbles.
How many marbles did Joe have in the beginning?

Compare:

Joe has 3 marbles. He has 5 more marbles than Tom.
How many marbles does Tom have?

From Riley et al, 1984

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Word problems in the final exam

1 **Sponsorløb på efterskolen** Opgave 1 giver højst 12 point

På Sofie og Hugos efterskole er der motionsuge. I motionsugen skal skolens elever deltage i et sponsorløb for at samle penge ind til en ny sportshal.

Eleverne har fået deres familier til at give et antal kroner pr. kilometer, de kan løbe på en time. Sofies familie vil give 60 kr. pr. kilometer, hun løber.

Foto: Opgavekommissionen i matematik

1.1 Hvor mange penge samler Sofie ind, hvis hun løber 10,5 km ved sponsorløbet?

Hugos familie vil give 45 kr. pr. kilometer, han løber.

1.2 Hvor mange kilometer skal han løbe for at samle 500 kr. ind?

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1 Sponsorship race at the school

There is an exercise week at Sofie and Hugo's school. During the exercise week, the school's students must participate in a sponsorship race to raise money for a new sports hall.

The students' families will give a certain number of kroner per kilometer they can run in an hour. Sofie's family will give DKK 60 per. kilometer she runs.

1.1 How much money does Sofie raise if she runs 10,5 km at the sponsorship race?

Hugo's family will give DKK 45 per. kilometer he runs.

1.2 How many kilometers does he have to run to collect DKK 500?

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The role of word problems in mathematics

1 Practice mathematical skills in "everyday situations"
- justification / motivation for mathematics

2 Develop problem-solving skills
- from simple problems to complex problems

3 Develop understanding of concepts
- e.g. understanding of the four basic operations

e.g. Verschaffel et al. 2020

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Why are word problems difficult?

General factors
- text elements (lix level, specific words and concepts)
- context (known / unknown, concrete / abstract)
- answer and question format

Specific factors
- number range (integers, fractions, etc.)
- the order of known elements
- semantic structure: different structure - same solution

- so, students should meet many types of word problems

e.g. De Corte & Verschaffel, 1987; Verschaffel et al. 2020

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Semantic structure, success and strategy

Difficulty level is a complex function of the semantic structure

Different semantic structure: Different types of errors

(e.g. Fuson 1992, Verschaffel & de Corte 1997)

The type of problem affects

- the student's success rate
- solution strategy

(De Corte & Verschaffel, 1987)

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Why are word problems difficult?

Simple (easy) word problems can be solved with informal strategies

that is, without formulating an abstract arithmetic equation

Complex (difficult) problems require transformation into abstract form

especially if technology is required

The difficulties arise in the transition from "simple" to "complex"

(e.g. Kieran 1992, Swafford & Langrall 2000)

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Our study

64 word problems:
14 addition, 23 subtraction, 14 multiplication and 13 division.

Each task was assigned a difficulty level:
easy or difficult.

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Examples, addition

Easy

Anna plays the piano 4 hours a week and Birgit plays the piano 6 hours a week.
How many hours do they play all together per week?

☒ 4 + 6 ☐ 6 − 4 ☐ 10 − 4 ☐ 4 · 6

Difficult

Frederik goes to 2 activities a week, that is 3 activities less than Salem.
How many activities does Salem go to?

☒ 2 + 3 ☐ 3 − 2 ☐ 5 − 2 ☐ 5 − 3

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Examples, multiplication

Easy

6 of the students in Viktor's class must each bring 15 cookies to the class party.
How many cookies do they take all together?

☐ 15 + 15 ☐ 15 : 6 ☐ 15 + 6 ☒ 6 · 15

Difficult

Carrots cost DKK 9 per kg.
How much does 2.5 kg of carrots cost?

☐ 18 + 2,5 ☐ 9 : 2,5 ☐ 9 + 2,5 ☒ 9 · 2,5

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Data

1081 students (86 classes, 31 schools) from 3rd to 6th grade participated.

The student's teacher scored each student's general level of mathematics on a scale from 1 to 5.

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Problem type			Difficulty type: Easy (0) Difficult (1)
Add_SF	+SF	combine and increase	0
Add_S	+S	comparison	1
Add_E	+E	after a reduction	1
Sub_R	-R	reduction	0
Sub_S	-S	comparison	1
Sub_O1	-O1	filling up, "right" order	1
Sub_O2	-O2	filling up, "wrong" order	1
Sub_E	-E	after an addition	1
Mult_G	*G	repeated addition	0
Mult_M	*M	amounts	1
Mult_A	*A	area	1
Div_L	%L	equal sharing	0
Div_M	%M	measuring	1

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Results - the general picture

Proportion of correct answers in general

- higher for addition than for the other problem types.

Proportion of correct answers (for all four basic operations)

- increased with increasing grade levels
- increased with increasing mathematics level
- decreased with increasing difficulty

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Results - the challenged students

Is there a relation between the proportion of correct answers and achievement level (teacher rating) and the degree of difficulty of the problem?

Or in other words:

Does academically challenged students generally find difficult problems, more difficult than other students?

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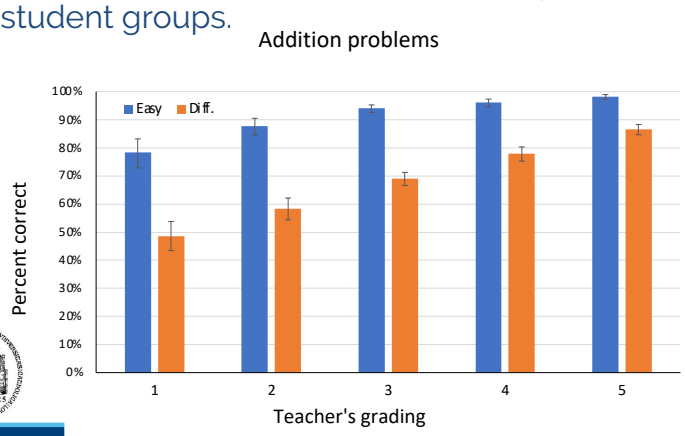
Results - the challenged students

Proportion of correct answers to easy and difficult problems divided into student groups.



For addition:

Larger difference between easy and difficult problems for challenged students.

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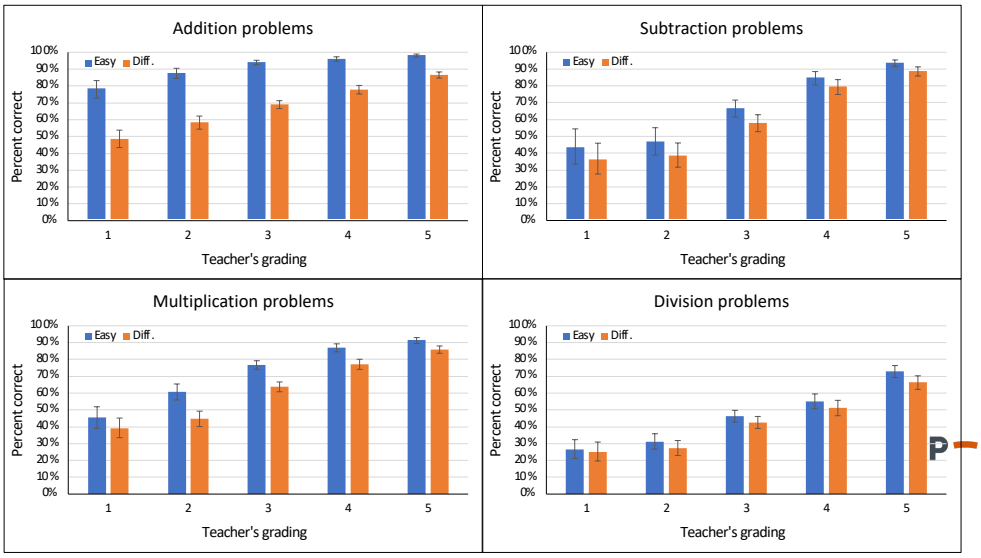
Results - the challenged students

For subtraction, multiplication and division the difference between easy and difficult problems are the same for all students.



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Conclusion - in general

Success rate in all four operations depends on student achievement level and type of operation.

The degree of difficulty of the basic operations: addition - multiplication - subtraction – division.

Difference in solution success for easy versus difficult problems largest for addition.

It seems:

For addition: The semantic structure (difficulty) has impact on the student's ability to choose a correct arithmetic expression.

For the other three types of operations: The semantic structure (difficulty) has little impact. It is probably about the operation type itself.

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Conclusion - the challenged students

Success rate <50% except for easy addition problems.

Greatest success with easy addition tasks (80%) - the rest is just hard!

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Implications for teaching

Do not just introduce one type of word problems within each basic operation.

Do not only give problems that are so easy, that they can be solved intuitively, without calculations.

Complement the easy problem with a focus on how to solve it with a calculator - and give support in this process.

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Roughly speaking, the challenged math students can only recognize quite simple addition tasks.

Or put another way:

For the challenged, the calculator has only one meaningful key among the four basic operations, the addition key:

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